Adatlap[1] témahirdetési javaslathoz a Csonka Pál Doktori Iskola Tanácsa részére

Témavezető[2] neve: Sipos András Árpád
e-mail címe[3]: siposa@eik.bme.hu

Téma címe (magyar és angol nyelven):
Weingarten-felületek analitikus vizsgálata, morfológiai és szerkezetmechanikai alkalmazásuk
Analytical investigation of Weingarten surfaces and their applications in morphology and structural mechanics

A **téma** rövid leírása[4] (magyar és angol nyelven):

A matematikában a minimálfelületek a később variációszámításnak nevezett eljárás hatékonyságának egyik első példáját adták. A felület funkcionál minimalizálása nem csak teszetős görbült felületeket eredményez, hanem egyben a sajtátfeszültséggel terhelt membránhéj alakját is megadja. Egyes összetettebb, a felület érintősíkjából kilépő terhek esetén az egyensúlyi felület ún. Weingarten felület, amelynél a felület minden egyes pontjában a H átlaggörbület és a K szorzatgörbület ugyanazon algebrai összefüggést elégíti ki. Ugyanakkor jól ismert, hogy a Weingarten felületek számítása a K szorzatgörbület jelenléte miatt egy erősen nemlineáris feladat.
A doktori kutatás célja a Weingarten felületek tulajdonságainak analitikus vizsgálata. Tekintve, hogy a klasszikus minimálfelület előállítható az átlaggörbület-folyam, mint geometriai parciális differenciálegyenlet (gPDE) stacionárius megoldásaként, célunk egy, a Weingarten felületeket számító algoritmus kidolgozása az ún. Bloore-folyam alkalmazásával. Kiemelt kérdés a folyam konveregncia tulajdonságainak vizsgálata és a tranziens állapot idejének becslése. Tekintve, hogy a Bloore-folyam az ütközéses kopást leíró gPDE, az analitikus vizsgálatok és a numerikus eljárások fejlesztése mellett további cél a természeti formavilág és különböző tehereloszlású membránhéjak alakja közötti hasonlóságok és különbségek feltárása.

In mathematics, minimal surfaces provided one of the first examples of the effectiveness of the theory, which later became known as the calculus of variations. Minimizing the surface area functional not only produces beautiful curved surfaces but also provides the shape of the membrane shell under eigenstresses. For loadings exiting the tangent plane of the surface, the equilibrium surface is a so-called Weingarten surface, where at each point of the surface, the mean curvature H and the Gaussian curvature K satisfy the same algebraic relation. However, it is well-known that the calculation of Weingarten surfaces is a highly nonlinear problem due to the presence of the Gaussian curvature K.
This PhD research aims to investigate the properties of Weingarten surfaces analytically. Considering that the classical minimal surface can be generated as a stationary solution of the mean curvature flow, i.e., a geometric partial differential equation (gPDE), we aim to construct an

[1] Az adatlapot aláírva és szkennelve a Doktori Iskola titkárának (Fehér Eszter, feher.eszter@epk.bme.hu) kell eljuttatni elektronikusan. A témahirdetés elfogadása esetén az adatlap felkerül a Csonka Pál Doktori Iskola (http://cspdi.bme.hu/felveteli/temahirdetesek), a témahirdetés rövid leírása pedig az Országos Doktori Tanács (http://www.doktori.hu/) honlapjára
[2] A témahirdetés elfogadása automatikusan a témavezető akkreditációját is jelenti az azévi felvételi eljáráshoz.
[3] Kérjük, olyan elérhetőséget adjon meg, ahová biztonsággal küldhetünk hivatalos értesítéseket.
[4] A téma rövid leírása (szóközökkel) 1000-3000 leütés hosszú. A jelentkező hallgatókat bővebben tájékoztató változatot, (mely a téma fent megadott releváns nemzetközi irodalmára tételesen hivatkozik) kérjük a mellékletben megadni.

algorithm for computing Weingarten surfaces using the so-called Bloore flow. A distinguished focus is the study of the flows's convergence properties and the estimation of the transient time. Considering that the Bloore flow is the gPDE associated with collisional abrasion, in addition to the rigorous analysis and the development of numerical methods, we also aim to explore the similarities and differences between the abraded forms in nature and the shape of membrane shells under various loadings.

A **téma** meghatározó irodalma[5]:

- Caffarelli, L., Sire, Y.: Minimal Surfaces and Free Boundaries: Recent Developments, Bulletin of the American Mathematical Society 57, pp. 91-106 (2020).
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A **téma** hazai és nemzetközi folyóiratai[6]:

- Bulletin of the American Mathematical Society (Sci)
- Differential Geometry and its Applications (Sci)
- European Journal of Applied Mathematics (Sci)
- International Journal of Solids and Structures (Sci)
- International Journal of Nonlinear Mechanics (Sci)
- Journal of Differential Geometry (Sci)
- Journal of the Mechanics and Physics of Solids (Sci)
- Proceedings of the Royal Society of London. Series A (Sci)
- Mathematics and Mechanics of Solids (Sci)
- Mathematical Geosciences (Sci)

A **témavezető** fenti folyóiratokban megjelent 5 közleménye:

- Michel, S., Sipos, AA.: Fragmentation of inflated elastic brittle rings: Emergence of the quasi-equidistant spacing of cracks, J. Mech. Phys. Sol. 179 Paper: 1005372, 17 p. (2023) DOI: 10.1016/j.jmps.2023.105372
- Sipos, AA., Várkonyi P. L.: The longest soft robotic arm, Int. J. Nonlin. Mech., 119 paper: 103354, 10 p. (2020) DOI: 10.1016/j.ijnonlinmec.2019.103354
- Sipos, AA: Ooid Growth: Uniqueness of Time-invariant, Smooth Shapes in 2D, Eur. J. Appl. Math. 31(1):172-282 (2020) DOI: 10.1017/S0956792519000019
- Sipos AA., Fehér E.: Disappearance of stretch-induced wrinkles of thin sheets: a study of orthotropic films, Int. J. Solids & Struct. 97-98: 275-283 (2016) DOI: 10.1016/j.ijsolstr.2016.07.021
- Domokos G, Gibbons GW, Sipos AA: Circular, Stationary Profiles Emerging in Unidirectional Abrasion, Math. Geosci. 46:4 pp. 483-491. (2014) DOI: 10.1007/s11004-013-9487-9

[5] Minimum 5, maximum 10 cikket vagy monográfiát kérünk felsorolni, amik között feltétlenül szerepelnie kell a legfrissebb, legismertebb eredményeknek.
[6] Minimum 5, maximum 10 folyóirat megadását kérjük, melyek között feltétlenül szerepelnie kell a PhD fokozatszerzés szempontjából elengedhetetlen (Scopus és/vagy Sci illetve Iconda) minősítésű idegen nyelvű folyóiratoknak is. Kérjük, ezeket a periodikákat a felsorolásban jelöljék meg.

A **témavezető** utóbbi tíz évben megjelent 5 legfontosabb publikációja:

- Sipos, AA.: About the number of hinges at failure of semicircular and pointed masonry arches, Math. & Mech. Sol. Közlésre elfogadva. (2023) DOI: 10.1177/10812865231196796
- Sipos, AA, Várkonyi P.L.: A unified morphoelastic rod model with application to growth-induced coiling, waving, and skewing of plant roots, J. Mech. Phys. Sol. 160 Paper: 104789, 14 p. (2022) DOI: 10.1016/j.jmps.2022.104789
- Sipos, AA., Domokos, G., Török, J.: Particle size dynamics in abrading pebble populations, Earth Surf. Dyn., 9:235-251. Paper: esurf-9-235-2021, 17 p. (2021) DOI: 10.5194/esurf-9-235-2021
- Sipos, AA., Domokos, G., Jerolmack, D. J.: Shape evolution of ooids: a geometric model, Scientific Reports 8 Paper: 6233 (2018) DOI: 10.1038%2Fs41598-018-19152-0
- Domokos G., Jerolmack D. J., Sipos A.A., Török Á.: How river rocks round: resolving the shape-size paradox, PLOS ONE, 9:2 Paper: e88657 (2014) DOI: 10.1371/journal.pone.0088657

A **témavezető** eddigi doktoranduszai[7]:
(név/felvétel éve/abszolutórium megszerzésének éve/PhD fokozat éve)

- **Fehér Eszter** 2014/2017/2019. (megj.: 2017-ben az EHBDT pályázatán második díjat nyert.)
- **Cao Siwen** 2020/./.
- **Sébastien Michel** 2020/./.

Melléklet: a téma bővebb leírása (angol[8] nyelven)

Budapest, 2024. január 29.

Témavezető aláírása

[7] Kérjük, a témavezetési tevékenységre vonatkozó adatokat abban az esetben is adja meg, ha témavezetőként a DI már korábban akkreditálta.
[8]A téma bővebb leírása angol nyelven csak akkor szükséges, ha a témavezető vállalja külföldi hallgató fogadását.

Analytical investigation of Weingarten surfaces and their applications in morphology and structural mechanics

András A. Sipos

1. Background

Minimal surfaces have a distinguished place in mathematics: they served as one of the first examples of the powerful technique we call variational calculus today (Meeks III et al. 2011). Minimizing the area functional delivers beautiful minimal surfaces (Colding & Minicotzzi 2006) and also delivers a solution for a problem in classical mechanics. The equilibrium shape under surface tension and fixed boundaries is also a surface with a minimal area; hence, it is a minimal surface. Soap films between wire boundaries realize such a shape. Rigorously speaking, a minimal surface associated with the parameter space $\Omega \in \mathbb{R}^2$ is characterized by

$$H(x) \equiv 0 \qquad \forall x \in \Omega, \tag{1}$$

i.e., the mean curvature H vanishes at all surface points. In the case of *generalized minimal surfaces*, the condition on the mean curvature is relaxed; in this case, for a fixed $c \in \mathbb{R}$

$$H(x) \equiv c \qquad \forall x \in \Omega, \tag{2}$$

holds. For example, soap films without boundaries, i.e., soap bubbles, realize this case.

Having a shell under a quasi-static load with a normal component, finding the equilibrium membrane surface, i.e., the shape that balances the external loads with in-surface tractions without bending, is a more delicate problem. Recent results of the literature (Rogers et al., 2003) demonstrate that under constant hydrostatic pressure, the membrane shape is a *linear Weingarten surface* (LW surface in the sequel) that follows the following linear relationship (Weingarten 1861, Weingarten 1863):

$$c_1 + c_2 H(x) + c_3 K(x) \equiv 0 \qquad \forall x \in \Omega. \tag{3}$$

Here c_1, c_2, c_3 are fixed constants, and $K(x)$ denotes the Gaussian curvature. Note that minimal surfaces ($c_1 = c_3 = 0$) and generalized minimal surfaces ($c_3 = 0$) are special cases of the linear Weingarten surface.

A recent publication (Tellier et al. 2023) demonstrated that the practical case, namely constant vertical loads on the curved surface, also leads to a linear Weingarten surface; however, in this setting, the rules of the so-called isotropic geometry (Sachs 1990) should be followed. It means that the values of H and K are computed based on the isotropic definition of the slope.

Although the definition of the linear Weingarten surface is straightforward, its computation is challenging, primarily because of the presence of the Gaussian curvature K. Similarly to the extensively studied Monge-Ampére equation (Dean & Glowinski 2003; Dean & Glowinski 2004; Beneamou et al. 2010), the *Weingarten operator* is highly nonlinear, as the maximal derivatives of the function representing the surface appear in the computation of K – regardless of the applied parametrization of the surface.

It is known that the minimal surfaces are steady-state solutions of the *mean curvature flow* (Caffarelli & Sire, 2020). In a similar fashion, linear Weingraten surfaces might be associated with the *Bloore flow*, which reads

$$v(t,x) = c_1 + c_2 H(t,x) + c_3 K(t,x) \qquad \forall x \in \Omega. \tag{4}$$

where $v(t,x)$ denotes the speed in the normal direction and c_1, c_2, c_3 are fixed, positive constants. This equation, introduced initially to explain the shape evolution of pebbles (Bloore 1977), is a nonlinear geometric partial differential equation (gPDE). Although the Bloore equation originates from the wish to describe collisional abrasion faithfully, it is also investigated in the mathematician community as one of the extensions of the classical *mean curvature flow.*

2. Research goals

We aim to contribute to further developing the theory of Weingarten surfaces, introduce algorithms based on the associated flows that compute those surfaces, and investigate the geometric features along the evolution. We also plan to contribute to the practical applications of LW surfaces (Pellis et al. 2021).

2.1 Local stability of the steady-state solutions of the unidirectional Bloore flows

Earlier studies (Sipos et al., 2011; Domokos, Gibbons, Sipos, 2014) show that shapes emerging in abrasion with a distinguished direction, such as bedrock abraded by grains transported by a fluvial stream, are converging to Weingarten surfaces. First, we aim to generalize earlier results on unidirectional abrasion in two dimensions into the three-dimensional setting. We show that a traveling wave solution must be a LW surface.

Q#1. Show that the traveling wave solution of the 3D Bloore flow, a LW surface, is locally stable. Compare traveling wave solutions of Euclidean and isotropic geometries.

2.2 Convergence properties of the unidirectional Bloore flows

Develop and implement an algorithm that establishes the Blorre flow and, for arbitrary (c_1, c_2, c_3) triples in eq. (4), compute the evolution until steady state. Identify the prerequisites of convergences. (What are the appropriate boundary conditions? What are suitable initial guesses?)

Q#2. Study the global attractivity of the traveling wave solution numerically. Compare the evolutions in Euclidean and isotropic geometries.

For practical applications, geometric properties, especially monotonous quantities, are essential to be monitored along the evolution. The classical 3D Bloore flow produces unexpected features (Domokos et al. 2014); hence, tracking the geometric properties and predicting the transient time are worthy of study.

Q#3. Describe the evolution of geometric features (e.g., number of umbilic points) of the surface along the evolution under the unidirectional Bloore flow. Introduce a model to estimate the time of the transient needed to find the vicinity of the steady-state solution.

2.3 Application of Weingarten surfaces in the design of membrane shells.

Considering that the Bloore flow is the gPDE associated with collisional abrasion, in addition to the rigorous analysis and the development of numerical methods, we also aim to explore the similarities and differences between the abraded forms in nature and the shape of membrane shells under various loadings.

Q#4. Explore the similarities and differences between the abraded forms in nature and the shape of membrane shells under various loadings.

Depending on the student's interest, all questions (Q1-Q4 or a subset), would be investigated during the four years of graduate study. Fluent English and expertise in algorithm development (preferably in Matlab and/or C++) are essential. Background in mathematical analysis is preferable.

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