Adatlap[1] témahirdetési javaslathoz
a Csonka Pál Doktori Iskola Tanácsa részére

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Téma címe (magyar és angol nyelven):

Repedés-morfológia a kontinuummechanika eszközeivel

Morphology of fracture via continuum mechanics

A **téma** rövid leírása[4] (magyar és angol nyelven):

Több lépcsős terhelés során a törés előtt kialakuló repedések a törési folyamat irreverzibilitása miatt érdekes lenyomatai a korábbi állapotoknak. A repedések kialakulásának és terjedésének modellezése a mechanika egyik kiemelten fontos és érdekes területe a XX. század kezdete óta. Jelen kutatás célja a terheléstörténet és a megfigyelhető repedések kapcsolatának elemzése a kontinuummechanikai eszközeivel.
Kizárólag rideg és kvázi-rideg anyagokkal (mint kövek és beton) foglalkozunk, az analitikus és numerikus vizsgálatokhoz a rideg törés variációs alapú megközelítését vesszük alapul. A vonatkozó irodalom hangsúlyozza, hogy ez a modell kiválóan alkalmas több repedés egyidejű, egymást befolyásoló növekedésének vizsgálatára. Jelen kutatás célja ezen állítás alátámasztása a korábban kifejlesztetett, irreverzibilitási feltételt tartalmazó egyenletek útkövetésére szolgáló eljárás segítségével. A kutatási terv néhány egyszerű problémát vet fel, ezek a variációs modellel jól vizsgálhatóak. A problémák elemzése új adatokat szolgáltat az elmélet továbbfejlesztéséhez, segíti a tartószerkezeti szakértőket a repedésképek elemzésében és a háromtengelyű feszültségállapot hatására keletkező vetők újszerű megközelítést nyújtja a szerkezetföldtan számára.

Cracking patterns arising during fracture under gradually applied loads are interesting markers of past (stress and strain) states due to the irreversible nature of fracture. Modeling of the formation and evolution of cracks has been a distinguished and interesting problem since the beginning of the 20th century in solid mechanics. This research aims to investigate the interplay between the loading history and the observable cracks via the methods of continuum mechanics.
We restrict our investigations to brittle and quasi-brittle materials (like rocks and concrete) and use the framework of variational brittle fracture for the analytical and numerical studies. Publications emphasize that variational brittle fracture is a perfect tool to model the evolution of multiple cracks that mutually influence growth. We aim to support this claim by applying a previously developed continuation code for governing equations possessing an irreversibility constraint. Some simple problems are collected in the research plan, which can be studied by variational brittle fracture.

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[1] Az adatlapot egy példányban *kinyomtatva és aláírva* a Szilárdságtani Tanszék titkárságára, *elektronikus változatban* pedig a Doktori Iskola titkárának (B.Kóródy Anna, korody@eik.bme.hu) kell eljuttatni. A témahirdetés elfogadása esetén az adatlap felkerül a Csonka Pál Doktori Iskola (http://www.szt.bme.hu/index.php/oktatas/csonka-pál-doktori-iskola), a témahirdetés rövid leírása pedig az Országos Doktori Tanács (http://www.doktori.hu/)honlapjára.

[2] A témahirdetés elfogadása automatikusan a témavezető akkreditációját is jelenti az azévi felvételi eljáráshoz.

[3] Kérjük, olyan elérhetőséget adjon meg, ahová biztonsággal küldhetünk hivatalos értesítéseket.

[4] A téma rövid leírása (szóközökkel) 1000-3000 leütés hosszú. A jelentkező hallgatókat bővebben tájékoztató változatot, (mely a téma fent megadott releváns nemzetközi irodalmára tételesen hivatkozik) kérjük a mellékletben megadni.

Analysis of these problems will provide new input for theoretical investigations, help structural diagnostics to explain observed cracking patterns and will provide a new approach for faulting under triaxial stresses in structural geology.

A **téma** meghatározó irodalma[5]:

- Francfort G.A., Marigo J.J. 1998. Revisiting brittle fracture as an energy minimization problem. *J.Mech. Phys. Solids*, **46** p. 1319
- Bourdin B., Francfort G.A., Marigo J.J. 2008 The variational approach to fracture. *J. Elasticity* **91** p. 5
- Lancioni G., Royer-Carfagni G. 2009. The variational approach to fracture mechanics. a practical application to the french *Panthéon* in Paris. *J. Elasticity* **95** p. 1
- Miehe C., Welschinger F., Hofacker M. 2010. Thermodynamically consistent phase-field models of fracture: Variational principles and multi-phase FE implementations. *Int. J. Num. Methods Eng.* **83** p. 1273
- Del Piero G. 2013. A variational approach to fracture and other inelastic phenomena. *J. Elasticity* **112** p.3
- Wu JY., Cervera M. 2016. A thermodynamically consistent plastic-damage framework for localized failure in quasi-brittle solids: Material model and strain localization analysis. *Int. J. Solids & Struct*. **88-89** p. 227

A **téma** hazai és nemzetközi folyóiratai[6]:

- International Journal of Solids and Structures (Sci)
- International Journal of Architectural Heritage (Sci)
- Journal of Elasticity (Sci)
- Journal of the Mechanics and Physics of Solids (Sci)
- Phsyica D: Nonlinear Phyenomena (Sci)
- Phys. Rev. E. (Sci)
- Phys. Rev. Lett. (Sci)
- Scientific Reports (Sci)

A **témavezető** fenti folyóiratokban megjelent 5 közleménye:

- András A Sipos, Eszter Fehér: Disappearance of stretch-induced wrinkles of thin sheets: a study of orthotropic films INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES **97-98**: p. 275 (2016)
- András A Sipos, Gábor Domokos, Douglas J Jerolmack: Shape evolution of ooids: a geometric model SCIENTIFIC REPORTS **8**: Paper 1758. 7 p. (2018)
- Healey TJ, Sipos AA: Computational stability of phase-tip splitting in the presence of small interfacial energy in a simple two-phase solid PHYSICA D: NONLINEAR PHENOMENA **261**: p. 62. (2013)
- Orsolya Gáspár, András A. Sipos, István Sajtos: Effect of stereotomy on the lower bound value of minimum thickness of semi-circular masonry arches INTERNATIONAL JOURNAL OF ARCHITECTURAL HERITAGE **12**: p. 1. (2018)
- Domokos Gábor, Kun Ferenc, Sipos András A., Szabó Tímea: Universality of fragment shapes SCIENTIFIC REPORTS **5**: Paper 9147. 6 p. (2015)

A **témavezető** utóbbi tíz évben megjelent 5 legfontosabb publikációja:

- András A Sipos, Emő Márton, László Fodor: Reconstruction of early phase deformations by integrated magnetic and mesotectonic data evaluation TECTONOPHYSICS **726**: p. 73 (2018)
- András A Sipos, Gábor Domokos, Douglas J Jerolmack: Shape evolution of ooids: a geometric model SCIENTIFIC REPORTS **8**: Paper 1758. 7 p. (2018)
- András A Sipos, Eszter Fehér: Disappearance of stretch-induced wrinkles of thin sheets: a study of orthotropic films INTERNATIONAL JOURNAL OF SOLIDS AND STRUCTURES **97-98**: p. 275 (2016)
- Domokos Gábor, Kun Ferenc, Sipos András A, Szabó Tímea: Universality of fragment shapes SCIENTIFIC REPORTS **5**: Paper 9147. 6 p. (2015)
- Domokos G, Jerolmack DJ, Sipos AÁ, Török Á: How River Rocks Round: Resolving the Shape-Size Paradox PLOS ONE **9**:(2) Paper e88657. 7 p. (2014)

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[5] Minimum 5, maximum 10 cikket vagy monográfiát kérünk felsorolni, amik között feltétlenül szerepelnie kell a legfrissebb, legismertebb eredményeknek.

[6] Minimum 5, maximum 10 folyóirat megadását kérjük, melyek között feltétlenül szerepelnie kell a PhD fokozatszerzés szempontjából elengedhetetlen (Scopus és/vagy Sci illetve Iconda) minősítésű idegen nyelvű folyóiratoknak is. Kérjük, ezeket a periodikákat a felsorolásban jelöljék meg.

A **témavezető** eddigi doktoranduszai[7]:
(név/felvétel éve/abszolutórium megszerzésének éve/PhD fokozat éve)

- **Fehér Eszter** 2014/2017/. (megj.: 2017-ben az EHBDT pályázatán második díjat nyert.)

Melléklet: a téma bővebb leírása (magyar és angol[8] nyelven)

Budapest, 2018. február 26.

Témavezető aláírása

[7] Kérjük, a témavezetési tevékenységre vonatkozó adatokat abban az esetben is adja meg, ha témavezetőként a DI már korábban akkreditálta.
[8] A téma bővebb leírása angol nyelven csak akkor szükséges, ha a témavezető vállalja külföldi hallgató fogadását.

Morphology of fracture via continuum mechanics

András A. Sipos

1. Background

As long as the well-documented elastic (or hyperelastic) models of mechanics is considered, we are constrained to the present or documented recent past states of a solid body. As far as the constitutive model is based on reversibility no clue about past states is preserved after their root cause (such as prescribed boundary displacements or external loads) is removed. Irreversible phenomena (such as creep or plasticity) make the analysis significantly harder, however these models can be in accordance with the second law of thermodynamics and imply a present state under the influence of former states (Howell et al., 2009). In other words, irreversibility opens the gate to draw some conclusions about the history of the stress and strain fields of a body by simply investigating its present state. This idea is widely used, among many other possible examples let us just mention the practice of diagnostics in structural engineering or fault-slip analysis in structural geology. In contrast to the wide range of possible applications, systematic analytical studies and their comparison against controlled measurements are rare, in most of the cases some simple mechanical hypotheses or human intuition is used for verification.

Here we aim to focus solely on one irreversible process: fracture of brittle and quasi-brittle materials which is refered to as *brittle fracture* in short. Brittle fracture produces rich and not easily predictable geometrical patterns of cracks in case of a sufficiently supported body under quasi-static loading. Investigation of brittle fracture started by the pioneering work of A.A. Griffith (Griffith, 1921), and his ideas gained a wide recent interest in the context of *variational brittle fracture* (Amor et al. 2009, Bourdin et al. 2008, Francfort & Marigo 1998). Although Griffith himself took a variational viewpoint, his model required *a-priori* knowledge about the path of a single crack. The regularized elliptic energy functional in variational brittle fracture is free from such a limiting prerequisite, furthermore a Γ-convergence result (Bourdin et al. 2008) demonstrated that in the zero limit the model is identical to Griffith's. Therefore, variational brittle fracture is a perfect approach to compute and analyze formation and evolution of cracks as long as the material behavior is dominantly brittle. We aim to investigate the interplay between the loading history and the observable cracks via the methods of continuum mechanics. During the investigations chemical reactions of the material are neglected thus to keep consistence with the laws of thermodynamics, healing of cracks is excluded.

The idea and rigorous judgment of variational brittle fracture originates in the mathematician community, although similar ideas (which focus directly on applicability) appeared in fracture mechanics, too (Bažant, 1998). Damage is associated with a (scalar, vector or tensor) state field. In the scalar case $\alpha = 0$ represents intact (elastic) material, $\alpha = 1$ corresponds to a completely damaged state. Here a *rate-independent* viewpoint is admitted, thus crack propagation occurs through a series of quasi-static equilibrium solutions of the governing equations as the load parameter is varied. Irreversibility implies α to be nondecreasing at any point of the domain. In terms of nonlinear programming, this requirement transform the problem into a *variational inequality*. Nondecreasing α implies that during the evolution the Karuch-Khun-Tucker (KKT) conditions must be fulfilled to have a local minimizer.

Numerical codes that handle full irreversibility are rare (Rabczuk 2013), in most of the cases global optimization with a frozen damage field over a threshold in α is applied. Let this approach be called *limited irreversibility*. Placing the problem into the regime of bifurcation theory can help to exceed this shortcoming. To establish equilibrium paths of the problem, based on the works of Poore (Poore 1987,1990) continuation of the equilibrium set is possible from a known state. A numerical method to exploit this idea and carry out continuation for problems involving irreversibility has been already developed.

2. Research goals

We aim to investigate simple problems which seems to be adequate to a detailed analysis, including experimental work.

2.1 Cracking of brittle materials

Q#1 Carry out simple, benchmark experiments to document evolution of crack patterns as the prescribed displacement along the edges of the specimen is increased. Compare the results against simulations based on variational brittle fracture.

For the experiments concrete and cut rock specimens with a size approximately 15x15x30-150 cm should be produced. With steel frames various fixed boundaries can be provided and controlled boundary motions along one of the sides can be applied in a displacement controlled manner. To carry out the cyclic loading (an unloading) the testing machine of the department laboratory is readily applicable. Formation of multiply cracks can be initiated by drilling holes into the specimen. The propagation of the cracking pattern during a load increment should be imaged by a high speed framing camera (1000-5000 fps).

2.2 Cracking patterns of a hemispherical dome

The investigation of equilibrium of a cracked hemispherical dome has attracted a significant interest since the middle of the 18th century, when Poleni validated that the cracked dome of the St. Peter's Basilica was safe (Heyman 1995). Being one of the oldest problems in engineering mechanics, surprisingly many papers have published even recently about equilibrium and safety of domes (D'Ayala 2001, Oppenheim 1989). The patterns of cracks under non-uniformly distributed, or even concentrated quasi-static loads is a much less researched area. Beyond the inspiration from structural engineering, the rotational symmetry involved makes it a perfect problem for mathematical analysis. The cracking pattern of a dome can be related to the wrinkling of an annulus made of thin films (Coman 2007, Davidovitch 2011): while the cracking pattern of the dome appears under tensile stresses, the wrinkling of the film is a buckling phenomena under compression. The wrinkle crests in this case arranged in a very similar, rotationally symmetric pattern as the cracking pattern of the dome. The significant difference between the two phenomena is irreversibility itself: for hyperelastic films (i.e. elastomers, such as polyurethane) the initial (unloaded, flat) state can be completely recovered. Nevertheless, the difference between their initial geometries requires extra care, thus the radial curvature of the reference state should be handled as a control parameter of the problem.

Q#2 A simultaneous study of the effect of irreversibility on the emerging pattern (of wrinkles and cracks, resp.) in case of an elastic annulus and a dome made of brittle material as it is gradually loaded. Is there any feature of the pattern that provides evidence about an irreversible underlying process?

This problem is perfect for an analytical study using classical methods from the calculus of variations and investigate the non-linear PDE-s in details. Nevertheless, this investigation will require a (likely FEM based) discretization and numerical continuation of the solution set. Continuation of problems with an irreversibility constraints is far not straightforward. The above mentioned continuation method can be readily modified to handle rotationally symmetrical geometries.
My final goal in this scope is to verify the numerical results against experimental data. The literature contains well documented experiments about the wrinkling of the annulus, but just with initially flat geometries. Experimental results for domes, especially measured geometries of the cracking patters are rare. Thus the second question here is an experimental one:

Q#3 Experimental investigation of the cracking of the dome under non-uniform distribution of the load. How the pattern reflects to repeated loading cycles?

This point aims lab experiments, too. Here the direct application of the results of the research is straightforward: we aim to analyze some historical, cracked domes, and show the applicability of variational brittle fracture in the field of structural diagnostics. A first step toward such an application is already published, there a single crack of the stones of the French *Panthéon* was analyzed (Lancioni & Royer-Carfagni 2009).

2.3 Cracking patterns in confined material

In the field of structural geology analysis of fault slip data is a main tool to separate deformation phases and model strain and stress history. Most of the stress inversion methods used to approximate the stress tensor from an observed set of faults and striae are based on the Wallace-Bott hypothesis (Wallace 1951, Bott 1959). It states that the stress tensor must have been oriented such way, that the maximal shear stress coincided with the direction of movement along the fault taken to be identical to the mean of the measured direction of striae on the surface of the fault. Although a statistical approach can help to overcome the shortcomings of the method (Sipos, 2018), there is still a general doubt about the validity of the Wallace-Bott hypothesis (Lisle 2013). Some results in elasticity (Tarantino 2013) support these critics. Furthermore, the problem of geometrical compatibility in case of triaxial loading is already noted by many researchers (Reches 1978, 1983). This debate should be placed into the context of fracture mechanics. Variational approaches of brittle fracture are perfect models to understand faulting and cracking patterns even in a geological scale. The first goal in this scope is to establish a simple 3D model to compute cracked surfaces (i.e. faults) under different levels of confinement stresses:

Q#4. Numerical simulation of faulting process using variational brittle fracture. Investigation of applicability and accuracy of the widely used stress inversion methods used in structural geology. Suggest a stress-inversion method based on the acquired data.

Depending on the interest of the student, either all questions (Q1-Q4), or a subset of them would be investigated during the four years of study. Fluent English, expertise in algorithm development (preferably in C++) is essential. Background in experimental work or mathematical analysis is preferable.

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